Introductory comments by the author*

This paper, certainly unorthodox, was submitted to the Canadian Journal of Mathematics at the invitation of Paulo Ribenboim, an associate editor at the time, but accepted only on condition that it be followed by one or more conventional papers that added some theorems in the customary sense to discussions of an otherwise strictly programmatic nature. For the reader unfamiliar with the recent history of L-functions and automorphic forms it is perhaps best to begin with a little personal background that is also, to some extent, a reflection of the concerns of a few mathematicians in the years just before and just after my professional beginnings.

Introduced as a student to the ideas of Hecke and of Selberg by Steven Gaal at Yale in 1959 / 60, thus to the notion of an L-function associated to a modular form, and to the theory of automorphic forms as a branch of functional analysis, I arrived at Princeton in the fall of 1960 where Salomon Bochner, with his customary generosity and enthusiasm, decided as soon as he was acquainted with my first, somewhat juvenile, results on Eisenstein series that I was a number-theorist, and did his best to make a silk purse of a sow's ear, ultimately insisting, although it was by no means customary for young, transient faculty members to be allowed to teach graduate students, that I offer a course in class-field theory, a subject then regarded as utterly arcane and about which I knew absolutely nothing.

The upshot was that I later brooded for some time (and thereby completely discouraged myself) on two topics that, so far as I knew, were quite distinct: that of attaching L-functions to automorphic forms and that of formulating a nonabelian class-field theory. I was certainly not alone, and there are traces of false starts in the literature.[1] The general solution ([L2]) to the first problem arose finally in a natural way from the theory of Eisenstein series (to the best of my recollection toward the end of 1966), and to my great surprise yielded upon reflection (over a period of months) a possible, and very persuasive, solution to the second ([L1]). This appeared to me then, and still appears to me today, an observation of the utmost importance: a coherent theory of L-functions attached to automorphic forms necessarily entails the solution of Artin's conjecture – as a part of *functoriality*. I add that a coherent theory, although this is something to be proved and remains a very difficult conjecture, does not give

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[1] The standard L-function, attached to autormorphic representations on forms of $GL(n)$, is exceptional. A good deal was known about it that now forms part of the basic theory. See [G] and [Tam] where references to earlier papers are given, as well as [GJ] which appeared later.

any new L-functions beyond the standard.[2] This is also an observation of central importance – again a part of functoriality.

I almost immediately communicated my discovery to André Weil, who did not understand (see the comment on p. 455, vol. 3 of his *Collected Works*) but who drew my attention to two important papers of his own [W1,W2]. From the first I learned about the Weil group, which could be immediately incorporated into the formalism I envisaged. >From the second I learned his formulation of a suggestion made several years earlier, in 1955, by Taniyama ([Pr]) and now famous because of its connection with Fermat's theorem. Taniyama proposed a relation between the Hasse-Weil zeta function of elliptic curves and the L-functions associated by Hecke to automorphic forms that is very similar to the relation between two-dimensional Artin L-functions and Hecke L-functions entailed by a coherent theory of automorphic L-functions.[3] Indeed over function fields the first implies the second, so that, in retrospect, it is astonishing that those familiar with Taniyama's proposal had not during the intervening years passed to Artin L-functions.

I was not previously in any concrete sense aware of Hasse-Weil zeta functions but once they were drawn to my attention two issues presented themselves. First of all, from the beginning the coupling of automorphic forms and Galois representations had for me both a global and a local aspect, so that I was eager to understand the local consequences of Taniyama's suggestion. One obvious paper to study was [Se], which suggested, upon comparison with the representation theory of $GL(2)$, at least one important conclusion: the special representation of $GL(2)$ over a local field, already familiar to specialists, corresponded to two-dimensional $\mathfrak{l}$-adic representations that were not completely reducible. A general form of this correspondence became clear as I learned more about $\mathfrak{l}$-adic representations from [D1].

It occurred to Deligne as well, who presumably was moving in the opposite direction. Even as a purely local statement the general form is difficult, and has since been proved for split groups by Kazhdan and Lusztig ([KL]). In general a Hasse-Weil zeta function or a motivic L-function is first identified as an automorphic L-function through a statement that the local factors of both Euler

[2] The term standard L-function as introduced in [L10] refers to those associated to $GL(n)$ as in [GJ]. This is a useful terminology as these appear to be the only indispensable ones. It has unfortunately been corrupted.

[3] The Taniyama suggestion, like the global suggestions of [L1], is to some extent, although the connection with modular forms is explicit, a stark statement that, in itself, does nothing more than identify two very differently defined Euler products, of which one can be analytically continued. It is applied, even proved ([Wi,TW]), in a richer context that includes the Eichler-Shimura theory and many other ideas.

products are equal almost everywhere, and the more precise investigation as to the relation between local components of the pertinent Galois representation on one hand and the pertinent automorphic representation on the other neglected. When it has been undertaken it has yielded results both appealing and useful. (See [Ca1,Ra1,Ra3,RZ,Z] as well as, for other reasons, [Wi,TW] and, when appropriate, the papers referred to in these articles.)

When Deligne suggested that we exchange our obvious responsibilities for the 1972 meeting in Antwerp and that I talk about the l-adic representations associated to modular forms and he about representation theory, I took advantage of the opportunity to establish that the suggested local correspondence for $GL(2)$ did in fact manifest itself inside the global correspondence. The result ([L3]) made for difficult reading, and even more difficult listening, but was necessary preparation for the present paper. I add that the result of [L3] was obtained with the help of an unproved lemma, later established by Illusie ([Il]), that anticipates – in a more elementary form – the theory of [GM1].

The second, more important, issue raised by Hasse-Weil zeta functions was whether the Euler products I had introduced were adequate to deal with them[4] Could every Hasse-Weil zeta function be expressed as a product of powers, integral but with arbitrary sign, of these Euler products? (The notion of motive was still far outside my ken.) In view of the Eichler-Shimura theory for modular curves, the obvious test cases were the varieties over number fields associated by Shimura in a series of papers during the sixties ([Shi1,Shi2]) to congruence subgroups and bounded symmetric domains.[5] It is well to be clear about the nature of the questions. To show that all L-functions associated to Shimura varieties – thus to any motive defined by a Shimura variety – can be expressed in terms of the automorphic L-functions of [L1] is weaker, even very much weaker, than to show that *all* motivic L-functions are equal to such L-functions. [6] Moreover, although the stronger statement is expected to be valid, there is, so far as I know, no very compelling reason to expect that all motivic L-functions will be attached to Shimura varieties. The L-function attached to an elliptic curve is of course expected, by the refined Taniyama conjecture that is already in part established by A. Wiles (in collaboration with R. Taylor),

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[4] The connection between Hasse-Weil zeta functions and automorphic L-functions thus appeared at a second stage, after the introduction of the L-group and the associated L-functions

[5] The term *Shimura variety* now current was, I believe, introduced by me, and so far as I know appeared in print for the first time in the paper under discussion. James Milne observes, however, that the term Shimura curve for the varieties of dimension one had appeared considerably earlier ([Ih]).

[6] If, as expected, all the L-functions of [L1] are ultimately equal to standard L-functions, then the question is whether all motivic L-functions are equal to standard L-functions. This appears, at first glance, simpler and therefore better. The difficulty is that, for Shimura varieties at least, it compresses two difficult questions, only one of which is at all accessible at present, into one, about which there is therefore nothing to be said.

not only to be equal to the L-function attached by Hecke to an automorphic form on $GL(2)$ but also to be a motivic L-function attached to a modular curve. In contrast the Artin conjecture, established for tetrahedral and octahedral representations, is obtained by showing that the corresponding L-function is equal to an automorphic L-function for $GL(2)$ that is not, in all probability, attached to a motive defined by a modular curve ([L6,Tu]).

I also stress that to show that the Hasse-Weil zeta function of a Shimura variety can be expressed in terms of the automorphic L-functions of [L1] is, at present no assurance that it can be analytically continued, even now, for the pertinent automorphic L-functions are not those accessible to the methods of [L2] or to other methods. In particular they are not known to be equal to standard L-functions.[7] The only comfort, and the purpose of the paper under discussion, is that if the coherent theory predicted by [L1] is valid, then at least for the arithmetic of Shimura varieties there is no need to look for further Euler products than those introduced there. The paper under discussion was only provisional, as indeed were all papers by the author on the subject. The general method, suggested in part by earlier papers of other authors, was described; some important obstacles were recognized; and were overcome for some simple examples.

A final point to be emphasized is that the various combinatorial problems arising in this paper and its continuations were much more difficult than I foresaw, and are still far from solved. It is very difficult to see how they could be treated, or even adequately formulated, outside the context of functoriality introduced in [L1].[8]

This is to some extent hindsight. During the academic year 1970/71, spent at the Universität Bonn, I began to try to penetrate Shimura's papers, whose purpose I recall was not to deal with zeta functions but simply to define the algebraic varieties associated to congruence subgroups acting on bounded symmetric domains as varieties over an appropriate number field. Understanding the papers was, and probably remains, no easy matter to someone with no experience in algebraic geometry.

[7] For $GL(2)$, thus for the usual Eichler-Shimura theory, the pertinent L-function is already standard, so that the problem of analytic continuation does not arise, or rather it had been solved by Hecke.

[8] The problem of extending, in some form, the Eichler-Shimura theory to the more general varieties investigated by Shimura was presumably quite early an obvious problem to those who were familiar with his papers. I have heard later and at second hand of one or two attempts that also used the method introduced by Ihara, in the context of curves, of counting numbers of points on the reduced variety and then comparing with a trace formula, but they appear to have been rudimentary. Without documents it is difficult to tell to what extent the important issues, whose resolution in whole or in part has cost considerable effort to several serious mathematicians, were recognized.

The critical insight came, fairly early in the course of the year, not from the papers themselves, but from experience with the discrete series, in particular with the work of W. Schmid on their cohomology ([Sc]), even though the pertinent information may not be an immediate consequence of his theorems.

To each Shimura variety is attached a connected Lie group G. The number of discrete-series representations of G with a given infinitesimal character is equal to the index ρ_G of the real Weyl group in the complex Weyl group and each has cohomology in the pertinent (middle) degree of dimension one. Since the automorphic L-functions defined in [L1] are attached to representations of a complex group ${}^LG$ associated to the group defining the bounded symmetric domain, no theory would be possible unless there was, for each G a representation r_G of the connected component of ${}^LG$ of dimension ρ_G. I found the representations by experiment, a clumsy method. When Deligne came to Bonn in the spring of 1971, and presented his version – a revelation to me in its clarity – of Shimura's theory (see [D2], where further references to Shimura's papers can be found; see as well various articles by Deligne and Milne in the general references), I discovered that the highest weight of r_G was already an integral part of it, and that it could be associated to a general form of what is called the congruence relation. Although, in contrast to that of a modular curve, the zeta function of higher dimensional Shimura varieties cannot be treated with the congruence relation alone, this was encouraging.

Such cohomological matters, to which harmonic analysis on real reductive groups is pertinent, were discussed toward the end of [L5] in an inchoate way. Sharper, clearer views can now be found in papers of Arthur and Kottwitz ([Ar1, Ko4]). Other issues are addressed in the present paper and three further papers [L7,L8,LL]. There are, at first glance, three important matters; the structure of the set of points on the reduced varieties; L-indistinguishability, later referred to as endoscopy; and the combinatorial arguments. There are reports on all of them in [Cor].[9]

The first was, in spite of my promises, not dealt with in any of my papers, but the necessary results were proven by Milne ([Mi1]). The problem of describing points on the reduced varieties turned out to be deep (for reasons not unrelated to those demanding the introduction of endoscopy), and was not solved for any large class of varieties until much later by Kottwitz and Reimann-Zink, and then by Wintenberger. There is a thorough treatment of the subject and its history in [Mi2].

[9] The collection [Cor] appeared soon after the paper. Two other collections that deal with subsequent developments due to J. Arthur, R. Kottwitz and others are [AA] and [Mon]. I am not in a position to do justice to many of the ideas and contributions of these mathematicians, and have, in the specific references, given only a representative group of papers that, all being well, will lead the reader to others.

As Kottwitz observed almost immediately, the combinatorial arguments of [L8] simply repeat (unwittingly) in a complicated and unrecognizable form those of [L6]. He pointed out that the combinatorial arguments for general Shimura varieties would just be forms of the fundamental lemma familiar from endoscopy ([L9,Ko1,Ko3]) and base change ([L6]). Endoscopy is a notion that arose in the study of Shimura varieties, where it appears almost immediately ([LL]), but conviction as to its validity was acquired by quite a different route, through the work of Shelstad ([She]) on endoscopy for real groups. Its value for the theory of automorphic forms, especially in the form of the much more sophisticated twisted endoscopy ([KS1,KS2]), is enormous, for – as originally suggested by H. Jacquet for the transfer from $SL(2)$ to $GL(3)$ – it is a powerful technique for establishing many important cases of the transfer predicted in [L1]. This is a program in itself that demands a deep study of harmonic analysis and the trace formula ([Ar4]) and that led, among other things to remarkable conjecture ([Ar1,Ar2]), in part accessible although not easily so, on the spectra of automorphic forms that has been partially verified by Moeglin and Waldspurger([MW,Mo1]); and to a closely related local conjecture that is now central to the problem of classifying unitary representations of real reductive groups ([ABV]) and of p-adic groups ([Mo2,V]).

The study of the zeta functions of Shimura varieties and of endoscopy has been brought very far forward by Kottwitz whose work is described in [Cl2]. Much more extensive results could be established were the fundamental lemma available in any generality. It has turned out to be remarkably stubborn, so stubborn that, as in [Ar4], it has sometimes been necessary to anticipate its demonstration in order to get on with other, also very serious, matters.

The fundamental lemma is intimately related to the existence of transfer for endoscopic groups. The existence of the transfer of functions from a reductive group to one of its endoscopic groups (defined in the context of functoriality ([L9])) is a problem of harmonic analysis that has to be preceded by appropriate definitions ([LS]). The fundamental lemma is a question about the realization of the endoscopic transfer in the context of Hecke algebras, and can be formulated independently of the question of existence of a general transfer although not of the definition of transfer factors. The initial treatments of some special cases of the fundamental lemma dealt with this formulation ([L6,Ko2,Rog]). Clozel ([AC]) introduced a method, exploiting the trace formula and usefully supplemented by important ideas of Kazhdan ([K]), that permitted the reduction of the fundamental lemma for general elements of the Hecke algebra to the unit elements, and then under the influence of Hales and Waldspurger the two problems – the fundamental lemma and the existence of the transfer – were fused. A great deal of progress has been made ([As,H1,H2,H3,HH,Wa1,Wa2,Wa3]), although a general treatment of what is at present a major outstanding problem in harmonic analysis is not yet available.

There are a number of problems arising from the study of the zeta functions of Shimura varieties that are not mentioned explicitly in the present article but whose solution has inspired important developments. First of all, the need to understand conjugation of Shimura varieties in order to deal with the Γ-factors attached to their Hasse-Weil zeta functions led to the Taniyama group: some of the definitions are in [L5]; many of the theorems are in [HC]; others are reviewed in Schappacher's article in [Mot]. Secondly, Shimura varieties appear at first as affine varieties, not as projective varieties, and their natural projective completions are usually singular, so that the treatment of their zeta functions with the help of the Lefschetz formula and the trace formula requires not only some consequences ([Ar3]) of the analytically difficult trace-formula for groups with non-compact quotient but also some form of the Lefschetz formula on singular varieties, anticipated in relatively simple cases in [L3, HLR, and BL], and in more difficult cases in [KR] and [R2], but appearing in the necessary generality only in work of Harder, Kottwitz, and Goresky-MacPherson that is only partially available at present ([GM1,GM2,GHM]). Related problems, for the usual cohomology rather than intersection cohomology, are treated in [La] where further references are given.

The analogue of the notion of a Shimura variety over a function field was introduced by Drinfeld ([Ma]) and it has undergone a parallel development. One important application by Laumon, Rapoport and Stuhler ([LRS,Ca2] is to establish, over the field F of formal power series over a finite field, the existence of the kind of correspondence between n-dimensional representations of the Galois group $\mathcal{G}(\bar{F}/F)$ and irreducible admissible representations of $GL(n,F)$ predicted in [L1].

Although there is little point in premature speculation about the form that the final theory connecting automorphic forms and motives will take, some anticipation of the possibilities has turned out to be useful. Motivic L-functions, in terms of which Hasse-Weil zeta functions are expressed, are introduced in a Tannakian context. A similar notion may be useful for the automorphic forms – among other reasons, in order to define the notion of the *support* of an automorphic representation – and one is suggested by functoriality, although in no very precise form ([L5]). The notion of the support of an automorphic representation is just one of the many difficult ideas that inform the development in [Ar4], where it has to be given, in the relevant context, sufficient precision to make it operational.

An adequate Tannakian formulation of functoriality and of the relation between automorphic representations and motives ([Cl1,Ram]) will presumably include the Tate conjecture ([Ta]) as an assertion of surjectivity. The Tate conjecture itself is intimately related to the Hodge conjecture whose formulation is algebro-geometrical and topological rather than arithmetical. I, however, have no idea what the exact nature of the relation between the two conjectures might be, nor which is primary and which secondary. Nonetheless it seems to me, for this and other reasons ([Cl3, Cl4]), that an examination of the geometric consequences of our present understanding of the relation between automorphic forms and the arithmetic of algebraic varieties, especially Shimura varieties, is an undertaking of some value.

As a final remark, I draw attention to the paper [Mu] that treats the relation between automorphic L-functions and the Artin conjecture suggested in [L1] in quite a different spirit, more analytic and less algebro-geometric, than that prevailing in these comments.

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